E 8645 (Pages : 4) Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2019

Third Semester

Complementary Course—Statistics—PROBABILITY DISTRIBUTIONS

(Common for B.Sc. Mathematics, Physics and Computer Applications. [Three Main])

[2013—2016 Admissions]

Time : Three Hours Maximum Marks : 80

Part A (Short Answer Questions)

Answer **all** *questions.*

1 *mark each.*

1. Write the p.m.f. of geometric distribution.
2. Write the moment generating function of a Poisson distribution with parameter λ.
3. Find the mean of Bernoulli distribution.
4. Define standard normal distribution.
5. Write the p.d.f. of uniform distribution over the interval (a, b).
6. What did you mean by fitting of a probability distribution ?
7. State the additive property of normal distribution.
8. Define Statistic.
9. Write the relationship between t and F distributions.
10. Write the mean of a t distribution with n degrees of freedom.

$(10 \times 1 = 10)$

Part B (Brief Answer Questions)

Answer any **eight** *questions.*

2 *marks each.*

11. What do you you mean by lack of memory property ?
12. Let X and Y be independent random variables having Poisson distribution with same parameter. Examine whether X – Y is a Poisson variable.

Turn over

13. State and prove the additive property of the binomial distribution.

14. Obtain the harmonic mean of a Beta distribution of first kind.

15. Derive the mean of the normal distribution, $N(\mu, \sigma^2)$.

16. Show that a finite linear combination of Normal random variable is a Normal random variable.

17. Derive the mean of the gamma distribution with p.d.f.

$$f(x) = \frac{m^p}{\Gamma(p)} e^{-mx} x^{p-1}, \quad x > 0.$$
$$= 0, \quad \text{otherwise.}$$

18. If X is distributed with mean 0 and variance 1, find the p.d.f. of X^2.

19. Find the least value of probability $p\{1 \leq X \leq 7\}$, where X is a r.v. with E (X) = 4 and Var (X) = 4.

20. State Bernoulli's law of large numbers.

21. Define *t*-statistic.

22. Write the p.d.f. of F distribution.

(8 × 2 = 16)

Part (Description/Short Essays)

Answer any **six** *questions.*
4 marks each.

23. Derive the recurrence relation for central moments of a Poisson distribution.

24. Derive the moment generating function of a binomial distribution with parameters n and p. Hence find its mean and variance.

25. Show that under suitable transformation on Beta distribution of second kind, one can obtain Beta distribution of first kind.

26. Find the distribution function of a random variable with p.d.f. given by :

$$f(x; a, b) = \frac{1}{b-a}, \text{ if } a < x < b$$
$$= 0 \quad \text{, otherwise.}$$

Also draw the graphs of p.d.f. and distribution function.

27. Derive the distribution of sample mean if samples are taken from $N(\mu, \sigma^2)$.

28. State and prove the additive properties of Chi-square distribution.

29. If X_1, X_2, X_3 and X_4 are independent observations from a univariate normal population with mean zero and unit variance. Find the sampling distribution of

$$V = \frac{3X_4^2}{X_1^2 + X_2^2 + X_3^4}.$$

30. Let $\{X_n, n \geq 1\}$ be a sequence of independent and identically distributed Poisson random variable with mean λ. Obtain the limiting distribution of $\sum_{i=1}^{n} X_i$ with the help of central limit theorem.

31. An unbiased coin is tossed 100 times. Show by using Chebyshev's inequality that the probability that the number of heads will be between 30 and 70 is greater than 0.93.

(6 × 4 = 24)

Part D

Answer any **two** *questions.*
15 marks each.

32. (a) If X and Y are independent Poisson variates with means λ_1 and λ_2 respectively.

Find the probability that (i) $X + Y = k$; (ii) $X = Y$.

(b) If X and Y are independent Poisson variates, show that the conditional distribution of X given $X + Y$ is binomial.

33. (a) Derive the moment generating function of $N(\mu, \sigma^2)$.

(b) Establish the recurrence relation between central moments of a binomial distribution.

34. (a) Let $X_1, X_2, \ldots, X_n$ be a sequence of independent random variables with $E(X_i) = \mu$ and $V(X_i) = \sigma^2, i = 1, 2, \ldots, n$. Define $\overline{X_n} = \frac{1}{n}\sum_{i=1}^{n} X_i$, the for any $\varepsilon > 0$, however small, prove that

$$P\left\{|\overline{X_n} - \mu| \geq \varepsilon\right\} \to 0 \text{ as } n \to \infty.$$

(b) Let $\{X_n\}$ be a sequence of mutually independent and identically distributed random variables with mean μ and finite variance σ^2. If $S_n = X_1 + X_2 + ... + X_n$. Prove that the law of large numbers does not hold the sequence $\{S_n\}$.

35. (a) State and prove the additive property of Chi-square distribution.

(b) If X_1 and X_2 are independent Chi-square random variables each with one degrees of freedom. Find λ such that $P\{X_1 + X_2 > \lambda\} = \frac{1}{2}$.

$(2 \times 15 = 30)$